Discrete Mathematics

Quiz # 3 (April 16, 2015)

Name:_____________________________ID:_________________________________

1. (30%) Prove that $(a * b) \bmod n = \big((a \bmod n) * (b \bmod n)\big) \bmod n$

Ans

Let $a = k_a * n + r_a$ and $b = k_b * n + r_b$

Thus $a \bmod n = r_a$ and $b \bmod n = r_b$ and then plugging into the left hand side

$\big((k_a * n + r_a) * (k_b * n + r_b)\big) \bmod n$

$= (k_a * k_b * n^2 + k_a * r_b * n + k_b * r_a * n + r_a * r_b) \bmod n$

$= \big((k_a * k_b * n + k_a * r_b + k_b * r_a) * n + r_a * r_b\big) \bmod n$

$= (r_a * r_b) \bmod n$

$= \big((a \bmod n) * (b \bmod n)\big) \bmod n$

2. (16%) Sort these growth functions from the slowest to the fastest growth without justification.

n^2, $n \log n$, 3^n, n, $n!$, $\log n$, 1, n^n

slow → fast							
1	$\log n$	n	$n \log n$	n^2	3^n	$n!$	n^n

3. The following is an algorithm for sorting a list of number $a_1, a_2, \ldots, a_n$.
   a. (30%) Use it to sort $7, 4, 9, 5, 3, -1$, and show $a_1, a_2, \ldots, a_n$ after each i iteration.
   b. (10%) In the worst case situation, how many times is the statement "*min* = *j*" executed given a list of number $a_1, a_2, \ldots, a_n$ as input?
   c. (14%) Find the smallest order big-O function for the number in (b) and justify your answer by the definition of big-O function.

```
procedure SelectionSort(a_1, a_2, ..., a_n: real numbers with n ≥ 2)
begin
    for i := 1 to n − 1
    begin
        min = i
        for j := i + 1 to n
        begin
            if a_j < a_min then
                min = j
        end
        exchange a_i with a_min
    end
end
```

Iteration (*i*)	$a_1, a_2, \ldots, a_n$ after each *i* iteration					
Initial	7	4	9	5	3	-1
1	**-1**	**4**	**9**	**5**	**3**	**7**
2	**-1**	**3**	**9**	**5**	**4**	**7**
3	**-1**	**3**	**4**	**5**	**9**	**7**
4	**-1**	**3**	**4**	**5**	**9**	**7**
5	**-1**	**3**	**4**	**5**	**7**	**9**

Ans

b.

$(n^2 - 1)/4$ **for odd** n

$n^2/4$ **for even** n

c. $O(n^2)$

$$\frac{(n^2-1)}{4} < \frac{n^2}{4} < n^2$$

By definition, $\exists c, k: \forall x > k: f(x) \leq cg(x)$**.**

We can find a solution: $c = \frac{1}{4}$, $k = 1$

So big-O function for the number in (b) is $O(n^2)$